

Twice random, once mixed: Applying mixed models to simultaneously analyze random effects of language and participants

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Abstract

This is a final draft of a paper accepted by and pending publication in *Behavior Research Methods*, projected publication date August 2011. For up-to-date information, see djmixed.googlecode.com.

Psychologists, psycholinguists and other researchers using language stimuli have been struggling for more than 30 years with the problem of how to analyze experimental data that contain two crossed random effects (items and participants). Classical analysis-of-variance does not apply, alternatives have been proposed but failed to catch on, and a statistically unsatisfactory procedure of using two approximations (known as F1 and F2) has become the standard. A simple and elegant solution using mixed model analysis has been available for fifteen years and recent improvements in statistical software have made mixed models analysis widely available. This paper aims to increase the use of mixed models by giving a concise practical introduction and by giving clear directions for undertaking the analysis in the most popular statistical packages. The paper also introduces the DJMIXED add-on package for SPSS, which makes entering the models and reporting their results as straightforward as possible.

When considering the design of an experiment that is to be analyzed with Analysis of Variance (Anova), there is a fundamental statistical difference between fixed factors and random factors. An informal definition of random factors is that they only test a subset of all possible levels of that factor and that there are no theoretical implications of the outcomes at each level of the factor. Participants are the most common random factor in psychology experiments: Only a subset of all available participants is tested and there is little or no theoretical interest in the performance of individual participants (of course, the individual participants should be inspected and screened, and if unexpected patterns of participant behavior are found this should be indicated).

Coleman (1964) and Clark (1973) realized that in all language experiments there are two random factors, participants and words. One could argue that this realization came too early for its own good: At that point in time, there was no fully satisfactory way to deal with two random factors in one Anova. Clark's suggestion to use F' or $minF'$ failed to catch on, despite evidence of its virtues (Forster & Dickinson, 1976; Wickens & Keppel, 1983; Santa, Miller, & Shaw, 1979, and others). In his paper, Clark also provided an alternative technique of using two approximate values, which was intended for reanalyzing existing experiments for which the raw data were no longer available. This method of using two approximate values, F_1 and F_2 , has become the *de facto*

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standard in the psycholinguistic literature.

Briefly refreshing well-known concepts will hopefully aid understanding (but the impatient reader can skip ahead to Example 1). The F test is a ratio of two variance estimates. It is the between-conditions variance divided by the within-conditions variance. The between-conditions variance is an estimate of the influence of the factor under scrutiny. The within-conditions variance (the *error term*) is an estimate of the noise in the data, at least in the simple case. We assume a significant effect of the condition if the F test indicates that the between-conditions variance is sufficiently larger than the error term, where sufficiency is determined by the F -distribution and the degrees of freedom.

The presence of two random factors causes the estimate of between-conditions variance to be contaminated by unwanted interactions. For a design with exactly one random factor, the unwanted interactions can be canceled out by choosing an error term that also contains those interactions. However, for a design with two random factors, no appropriate error term exists (Clark, 1973, see also Baayen, Tweedie, & Schreuder, 2002; Jackson & Brashers, 1994; Keppel & Wickens, 2004, p. 539). Hence, no algebraically exact F test can be computed for these designs, if one stays within the framework of Anova.

The solutions for this problem that have been proposed in the psycholinguistic literature before can be divided into four strands: First, some propose using the F' (or $\min F'$) test, a test that constructs an approximate error term (Forster & Dickinson, 1976; Maxwell & Bray, 1986; Santa et al., 1979; Wickens & Keppel, 1983). Second, some propose to do two analyses, F_1 and F_2 . In each of these analyses, one random factor is analyzed and the other is treated as fixed, leading to two approximate tests which should then be evaluated together

(Wike & Church, 1976). Despite its obvious shortcoming of being based on incorrect assumptions, this method has become standard practice. Third, some have argued that, at least in certain experimental settings, items could be classified as a fixed factor, by-passing the problem all together (Raaijmakers, Schrijnemakers, & Gremmen, 1999).

This paper favors a fourth solution which is based on *mixed modeling*, a technique for combining random and fixed factors into one analysis which has been developed since the 1980's. The name mixed modeling refers to mixing random and fixed effects, but the same technique is also known under other names, such as cross-classificational hierarchical linear models (Raudenbush, 1993). This technique bypasses all the problems that the classical Anova has, making it possible to present a very simple and straightforward analysis of a design containing two random factors.

This fourth solution has a number of separate origins, most of which are outside of psycholinguistics. The mixed, crossed effects model that is the key to this solution was used in Raudenbush (1993) and further extended by Rasbash and Goldstein (1994). An independently created application to Item Response Theory (IRT), including predictors at the participant level, has been proposed by Mislevy (1987) and others (see also Rijmen, Tuerlinckx, De Boeck, & Kuppens, 2003, on the relationship between IRT and mixed models). Baayen et al. (2002) were probably the first to publish on it in the context of psycholinguistic experimentation.

Mixed models and hierarchical models have been popularized in a number of recent publications: For example, Cheng, Sheu, and Yen (2009) have demonstrated the use of subject-specific random effects in the expectancy-valence

model of the Iowa Gambling Task. For the analysis of eyetracking data, Barr (2008) has made a case for using hierarchical logistic regression. Lee and Vanpaemel (2008), Rouder and Lu (2005), and Shiffrin, Lee, Kim, and Wagenmakers (2008) have written on the general applicability of hierarchical Bayesian methods for building and comparing models of cognitive tasks. The current paper aims to popularize the mixed model under psychologists dealing with language materials.

Whether a factor is treated as random or fixed is, to a certain extent, up to the researcher (Jackson & Brashers, 1994). The choice has implications for the generalizability of the findings, for the type of statistical questions that can be asked, for the fit between data and the model, and for the conceptual match between the model and the theory. There are various ways to define random factors (or more generally speaking, random effects or random coefficients). In the Hierarchical Linear Modeling (HLM) literature, random effects are introduced to account for the relatively high correlation between data points that fall within one hierarchical level (i.e., students in one class are more alike than students across classes). From a Bayesian perspective, random factors are introduced to increase generalizability and accuracy. From a classical (frequentist) point of view, random factors allow for more parsimonious models with fewer parameters.

Without a good knowledge of the statistical underpinnings, it may be hard to determine which factors are best treated as random in a psycholinguistic experiment. A helpful guideline is whether the levels that are tested are of direct theoretical relevance. Consider an experiment comparing the efficacy of complex and simplex primes in a lexical decision task. The levels of the fixed factor PrimeType (simplex and complex) are determined by and directly

related to the theoretical question under consideration. The random factor Words is different: Even if words are carefully selected to match on many dimensions, the actual words that are selected for the two levels of PrimeType do not affect the conclusions drawn from the experiment. In addition, there is no theoretical interest in the (average) priming effect exhibited by individual words, whereas the average priming effects of the PrimeType levels will be the main finding of the experiment. As a rule of thumb, one can look at the reported means: Papers invariably include a table listing the average RT for each level of each fixed factor in the experiment, whereas levels of the random factors are not reported or delegated to an appendix. (see also Jackson & Brashers, 1994 for an in-depth discussion of random factors).

As defendants of the third solution on how to combined random items and participants, Raaijmakers et al. (1999) suggest to analyze items as fixed in those experiments that basically deplete the pool of possible words. In that case, they argue, items are not selected *at random*. However, random selection of words is a sufficient but not a necessary condition for treating items as a random factor (Jackson & Brashers, 1994; Raudenbush & Bryk, 2002). Even when (nearly) all possible items were used because of multiple selection restrictions, the choice of stimuli has little or no theoretical repercussions and items should be treated as random to allow generalization of the findings beyond the set of items used in the experiment. Raaijmakers et al.'s argument would hold for a hypothetical experiment in phonology, in which the pronunciation time for the words *goat* and *goal* are compared to *coat* and *coal*. Here, the theoretical interest lies in the pronunciation times for these actual four words and items can be treated as fixed.

The textbook Anova is limited to one random factor because of restrictions in

its underlying linear model and the way this underlying model is computed. Another limitation of the textbook Anova is that fixed factors and random factors are treated very similarly, despite their apparent differences. The mixed model analysis overcomes both limitations: It allows for more than one random factor in the design and it treats random factors inherently different from fixed factors. The underlying linear model of the mixed model is different and can often only be solved by computer intensive iteration and approximation techniques. To a researcher using a modern computer, this technical difference is irrelevant. Mixed models have additional benefits such as that they can naturally handle unequal numbers of observations in the cells of the design.

Mixed models are closely related to Hierarchical Linear Models (HLM). The emergence of hierarchical linear modeling has transformed statistical practice in many areas of the social sciences over the past 15 years (Raudenbush & Bryk, 2002) and has been available, in rudimentary form at first, in SPSS since version 11. Mixed models differ from HLM in that they do not require a *hierarchical* relationship between the factors. In fact, HLM can be viewed as a special case of mixed modeling. This makes mixed models well-suited for language research: It is hard to convincingly argue that Items are nested under Participants or that Participants are nested under Items (but see Richter, 2006, for a defense of the former). In almost all experiments Items and Participants are crossed as every participant will see each item or every participant will see one variant of each item, as has been previously argued by Baayen, Davidson, and Bates (2008), Quené and Bergh (2008), and others.

The fundamental difference between a textbook linear model (also called classical linear model or ordinary least-squares linear model) and a mixed model is the presence of random effects in the model. To see how random effects are

represented, I will first revisit the representation of fixed effects.

Consider the following simplistic standard linear model of the simple experiment outlined above:

$$Y_j = \beta_0 + \beta_1 \cdot PrimeType_j + \epsilon_j$$

Here, PrimeType is dummy coded (simplex is zero and complex is one) and j indexes the different observations. This formula says that the observed reaction time can be modeled as the sum of the intercept (β_0), an influence of the variable PrimeType quantified by β_1 , and error. Because of the dummy coding chosen, the value of PrimeType is zero for simplex words, so the intercept (β_0) will be the *expected* reaction time for all simplex words. For the complex words, PrimeType equals one and an additional value (β_1) allows complex words to have a different expected RT from simplex words: The expected reaction time for complex words is $\beta_0 + \beta_1$. The error term ϵ_j is taken from a normal distribution with mean zero, which allows actual observations to differ from predictions.

If the experiment had four conditions instead and those conditions were treated as a fixed factor, the formula would look like this:

$$Y_j = \beta_0 + \beta_1 \cdot PTB_j + \beta_2 \cdot PTC_j + \beta_3 \cdot PTD_j + \epsilon_j$$

Here the four levels of PrimeType are dummy coded in three variables, PTB to PTD. The variables β_1 to β_3 will represent the reaction time differences between level B to D and the comparison level A, as shown in Table 1.

== TABLE 1 ==

This is a variant of the items-as-a-fixed-effect model. There are two statistical

complications with this model: First, when the number of levels of a fixed factor is increased, the model becomes more complex. Second, because the β terms are model parameters (they determine the content of the statistical model), any conclusions drawn from this experiment are strictly speaking conditional upon the values of β that were observed. This is less surprising than it may sound: If a simplex versus complex difference of 100ms was observed, the conclusion drawn from the first model would be that future research will also find a 100ms difference. If the observed difference (β_1) is different, the prediction changes. Therefore, the predictions of the model are conditional on the values of the model parameters.

It follows that if the factor Item with k levels (items) is modeled as a fixed factor, $k - 1$ dummy variables and $k - 1$ corresponding β s are included in the model formula. When interactions between Item and another dummy coded (categorical) factor R are included in the model, another $(k - 1) \times (r - 1)$ terms are required (where r is the number of levels of R). Clearly, applying the fixed factor approach to items can lead to very complex models: If there are 32 items in 4 conditions, $(32 - 1) \times (4 - 1) = 93$ different β terms are required. Despite its complexity this model would not constrain the values of the various β s at all. An experimenter would reasonably expect that items within one condition have similar average reaction times (β s), but no statistical property of this items-as-a-fixed-effect model enforces this.

Whereas β_0 and β_1 are parameters of the model that represent one single value (e.g., a 500ms intercept, a 100ms condition effect), the error term ϵ is a vector parameter that represents as many values as there are observations: There is one value ϵ_j for each observation j . In statistical output, the individual values ϵ_j are normally not listed, but the variance of all values is reported as the

variance of ϵ , denoted s_ϵ^2 or error variance. Mathematically, the values of the vector ϵ are modeled by a known statistical distribution with a certain mean and variance. The usual assumptions for ϵ are that its shape is that of the normal (Gaussian) distribution with a mean of zero and a variance that is a model parameter, the error variance s_ϵ^2 . In terms of conditional inference, this means that the conclusions drawn from this model are only conditional on the variance of the error term and on the fact that the errors should be normally distributed, and not on the actual error values that were observed. In other words, the conclusions from one experiment hold for all future experiments in which a similar error variance is obtained.

Vector parameters are used in mixed models to model random effects in a way that is quite similar to the treatment of residuals in the classical model. Instead of assuming a different β for each level of the factor Item, the levels are modeled by a vector parameter u , which has a different value for each item i . A value u_i reflects the relative speed of item i compared to the prototypical item in that condition. For an average item, u_i is close to zero as the average item is close to the prototypical item. For a slow item, u_i is relatively large and positive, increasing the expected RT. For a fast item, u_i will be relatively large and negative, reducing RT.

In other words, the item-specific value u_i adjusts the expected reaction time to reflect the relative speed of item i . The conceptualization of u as a vector of *item-specific adjustments* to the modeled reaction times has shown to greatly aid understanding of mixed models. Quite intuitively, all adjustments u_i are assumed to center around a mean according to a normal distribution with a certain variance, as shown in Figure 1: In the left panel, each line represents the effect of priming on one (hypothetical) item. The priming effect is constant,

but some items are inherently faster or slower than others, which leads to a distribution of lines. In the right panel, the distribution of item adjustments around the overall item mean is shown, which is close to normal. This means that the items within one condition are modeled as necessarily similar to each other, with the majority of items having an adjustment that is close to zero. Outlier items are possible, but they should be less likely the further they are removed from zero.

== FIGURE 1 ==

Returning to the experiment with simplex and complex words, the expected reaction time for a morphologically complex item i will have three parts in a mixed model: The intercept β_0 , the effect of condition β_1 , and the *adjustment* specific to this item u_i . This leads to the following mixed model formula (which does not include any effects related to participants yet)

$$Y_{ij} = \beta_0 + u_i + \beta_1 \cdot PrimeType_i + \epsilon_{ij}$$

with $u \in \mathcal{N}(0, \sigma_u^2)$ and $\epsilon \in \mathcal{N}(0, \sigma_\epsilon^2)$.

Here, the j -th observed reaction time on item i is modeled as the intercept β_0 , a random effect (relative adjustment) for this i -th item which has strength u_i , an effect of the PrimeType (simplex vs. complex) which has strength β_1 , and a residual value specific to this observation. The values of vector u are taken from a normal distribution (\mathcal{N}) with mean zero and variance σ_u^2 , making u similar in many ways to residual vector ϵ , which is taken from a normal distribution with mean zero and variance σ_ϵ^2 . Because the average values of u_i and of ϵ_{ij} are both zero (see Figure 1), the *expected* reaction time for any item is based

on the intercept and the effect of PrimeType only:

$$E(Y) = \beta_0 + \beta_1 \cdot \text{PrimeType}$$

Although a separate value u_i for each item is computed, only one model parameter is used. This model parameter represents the variance between items, s_u^2 . The parametrization on the variance implies that the conclusions drawn from this model are conditional only on the observed variance between the words and the fact that they are approximately normally distributed with mean zero. In practical terms, the conclusions drawn from this experiment should hold for all future experiments in which a similar item variance is obtained. An items-as-a-fixed-effect model above would be conditional on the actual effects found for individual words.

Modeling the factor Item with a random effect u in a mixed model has a number of interesting conceptual implications, when compared to the item-as-a-fixed-effect model discussed above: Similar to the earlier guideline criterion for random factors, modeling the values as a distribution in the mixed model agrees with a limited theoretical interest in specific values for each item. Second, the generalizability of the model with a random effect is greater, because the conclusions are only conditional on the variance of all words (s_u^2) and not on number and values of the individual β weights, as in the item-as-a-fixed-effect case. In the mixed model, the length of the vector u is not a model parameter, whereas the number of β weights in the fixed effects model changes if more items are added. Because the effects of individual words are modeled as taken from a normal distribution, the mixed model assumes that most words are similar, centered around a prototypical or ideal word, with outlier words becoming less frequent as they get further removed from the

average. If Item is modeled as a fixed effect, there are no constraints on the values of the individual item effects (β weights) at all. Because a mixed model estimates the model means and parameters using a precision weighted average (Raudenbush & Bryk, 2002, p 40, for details), the number of observations per participant and per item can vary substantially without any repercussions for the analysis. Finally, as will be outlined in more detail below, testing the mixed model does not depend on approximating F-values, which allows for a larger and more diverse set of statistical questions that can be answered.

One note of caution is in place here: Statistical inference from one model to a second set of data is, strictly speaking, conditional on the actual value of each parameter of the model: An item-as-a-fixed-factor model based on 20 items cannot be extended to a dataset with 21 items. In practice, researchers would use inductive reasoning to argue that extending the model to the second dataset is reasonable: The similarity between the two data sets and the lack of evidence indicating that one extra item may drastically change the outcomes will convince most readers that this is appropriate. The critical consideration here is that a mixed model approach does not require inductive logic in this case as the number of items is not a model parameter and the model can be applied without reservations.

We have so far only dealt with the effect of the random factor Items. Below the example will be extended to include a random factor Participants. A commonly raised question is how this mixed model analysis (and especially one with a random factor for Participants) compares to a repeated measures Anova. The difference between the two approaches is that in mixed models, Items, Participants and the random variation between individual Items and Participants are included predictors in the model. A repeated measures

analysis, on the other hand, only includes a predictor that captures the differences between participants' average RTs. With this, it can partition the sum of squares into three parts: within-condition variance, between-condition variance and between-participant variance. The additional third source of variance sets it apart from normal Anova and reduces the error term (within-condition variance) so that a more sensitive F value can be computed.

Example 1

After this theoretical and conceptual overview of how mixed modeling works and how it accounts for random factors in the data, a number of examples will be presented that will be analyzed in SPSS. Both the standard SPSS MIXED syntax and the use of the SPSS extension package DJMIXED will be discussed. Matching syntax for SAS and the free statistical package *R* is supplied in the on-line appendix.

The procedure and implications of using a mixed model analysis will be demonstrated from an example dataset containing priming data obtained from 34 subjects. Each participant made a lexical decision on 62 experimental items, for a total of 2004 valid data points (5% missing data). The two factors of interest were Priming (the critical word was the first word of a pair, priming absent, or the second word of a pair, priming present) and Morph (the critical word was part of an inflectional or a derivational pair). Other properties of the items that may influence the reaction time were matched. All stimulus words were part of 31 triplets formed by a base word, one of its inflections, and one of its derivations. Each participant first saw either a derivation or an inflection, followed by the matching base word. As shown in Table 2, there is

an indication of an interaction between Priming and Morph but the standard deviations of the cell means are sizable. (This is a real data set, in which I artificially strengthened the effect of Morph for didactic purposes).

== TABLE 2 ==

For comparison, an F_1/F_2 based analysis, using repeated measures in the F_1 , resulted in the following mixture of significances. The factor Priming is significant by F_1 and by F_2 : $F_1(1, 33) = 74.6, MSe = 2609, p = .000$; $F_2(1, 30) = 77.4, MSe = 2317, p = .000$. The factor Morph is significant by F_1 and by F_2 : $F_1(1, 33) = 18.3, MSe = 1013, p = .000$; $F_2(1, 30) = 14.9, MSe = 1204, p = .001$. The interaction between Morph and Priming is significant in F_1 but not in F_2 : $F_1(1, 33) = 9.4, MSe = 860, p = .004$; $F_2(1, 30) = 3.9, MSe = 2200, p = .058$.

Below, arguments for using a slightly more complex approach will be outlined, but it is instrumental to see what a very straightforward mixed model for this data looks like. In the terminology introduced above, the mixed model will contain item-specific *adjustments* to the predicted reaction times, to model that some items are easy (negative adjustments) and some items are hard (positive adjustments). The set of all item-specific adjustments is modeled by a normal distribution with a mean of zero, which has two consequences: The average adjustment is zero and larger adjustments should be less frequent than small adjustments.

In addition to the item-specific adjustments, we will also introduce participant-specific adjustments in this model. These adjustments are drawn from a second, independent normal distribution and they model that some participants are fast (large negative adjustments) and some participants are slow (large

positive adjustments), but most participants are close to average (have an adjustment which is close to zero).

We have to extend the notation introduced above to incorporate item-specific adjustments (u_{0i}) and participant-specific adjustments (u_{0p}). The letters i and p indicate the type of adjustment, the zero will be used later. The resulting model is identical to the corresponding classical regression or Anova model, but for the inclusion of the two random effects.

Mixed Model 1 A mixed-model was fitted to the data that contained the fixed effects of Priming, Morph, and their interaction, and two random effects accounting for participant-specific and item-specific adjustments to the Intercept. The mixed model formula is

$$Y_{pi} = \beta_0 + u_{0p} + u_{0i} \\ + \beta_1 \text{Priming}_i + \beta_2 \text{Morph}_i + \beta_3 (\text{Priming}_i \times \text{Morph}_i) + \epsilon_{pi}.$$

This model claims that observed reaction times can be modeled with a general intercept term β_0 , which is modified by a participant-specific adjustment u_{0p} (which distinguishes fast from slow participants) and an item-specific adjustment u_{0i} (which distinguishes fast from slow items). The expected reaction time is further modified by the effect of Priming (of strength β_1), the effect of Morph (of strength β_2), and their interaction (of strength β_3). Finally, the observed reaction times differ from the predicted reaction times by an observation-specific amount of error, ϵ_{pi} (error is observation-specific because each participant sees each item only once).

Because both Priming and Morph are dummy coded, the interaction effect β_3 applies only to the observations for which Morph equals 1 and Priming equals

1. The presence of a significant interaction term tells us that the *combined* effect of Morph and Priming is different from the sum of their effects ($\beta_1 + \beta_2$).

The results of this model are summarized in Table 3. The fixed effect of Priming was highly significant, $F(1,72)=47.9$, $p=.000$, the effect of Morph reached significance, $F(1,184)=9.2$, $p=.003$, and so did the interaction, $F(1,174)=4.2$, $p=.042$. Both random effects were significant: for u_{0p} , $Z=3.90$, $p=.000$; for u_{0i} , $Z=4.27$, $p=.000$, showing that their inclusion was warranted.

One could clearly argue that a mixed model analysis is easier to report and easier to understand than the matching F_1/F_2 analysis, as there are simply fewer F tests. The significance of the random effects u_{0i} and u_{0p} should be reported but they are not of direct theoretical relevance. The significance of u_{0i} and u_{0p} effectively mean that items differed from each other and participants differed from each other, which is to be expected in any experiment. In fact, absence of significance should be discussed in more detail: If the effect of u_{0i} were non-significant, this could mean that items are almost identical, which is unexpected and may be theoretically interesting.

To stay within the classical Anova report, two degrees of freedom were reported for each F-test. However, for mixed models the denominator degrees of freedom does not correspond to the number of cases or items, but it is computed in a different way (via the Satherthwaite method). The numerator degrees of freedom is identical to the number of levels of the factor minus one, as usual.

== TABLE 3 ==

Although the model formula may look complex, the matching SPSS syntax is very simple when using the SPSS extension module that was specifically

written for this paper (the extension module can be downloaded from the journal's archives and from djmixed.googlecode.com). Figure 2 shows the graphical interface to the DJMIXED package, while Figure 3 shows the corresponding DJMIXED syntax: The part that spans lines 2 to 7 defines the mixed model discussed here. Most parts should be self-explanatory. The PREDICTORS statement (line 4) simply lists the factorial predictors and their interactions. Currently, co-variates (interval-level predictors) cannot be included in DJMIXED. The NAME statement (line 7) is used to give the model a name. Names should be enclosed in quotes but can otherwise be freely chosen.

== FIGURE 2 ==

The second block of code, lines 8 to 10, prints out a summary table of the named model. The table is very similar to the one shown in Table 3. The output of the MIXEDMODEL subcommand (and the underlying SPSS MIXED command) is quite verbose, so this summary table can help users find the relevant numbers quickly.

To help the user keep track of the voluminous output, the first command has an extra option OUTPUT. If this is set to 'split' (the default), the full analysis results are directed to a secondary output window and the model summary is automatically generated in the main output window. The other two possible values are 'full' (no secondary window is used, the full SPSS output is shown without summary) and 'none' (no output is generated at all).

== FIGURE 3 ==

Similar output can be generated with plain SPSS commands: The DJMIXED package prints out the equivalent syntax every time it is run. The plain SPSS

syntax for all commands used here is listed in the online appendix and, for this model, shown in Figure 4. When not using DJMIXED, the output cannot be directed to two windows and the model summary is not available (and neither is the model comparison that we will encounter later).

== FIGURE 4 ==

Example 2, stepwise analysis

The analysis presented above is a significant statistical improvement over the double approximation via F_1 and F_2 and the other approaches mentioned earlier. For most purposes, this analysis should suffice. For the interested reader, a statistically more thorough exploration of the significance of the Priming by Morph interaction can be made by presenting a stepwise analysis.

A step-wise mixed model analysis is very similar to a forward-stepping linear regression analysis: Starting with a very simple model, additional model terms are introduced until the point that model fit is no longer improved. A step-wise mixed model analysis should start with a model (often called the *null model*) that contains the random effects of participants and items, but no other predictors (Model 2, introduced below). In the next step, the fixed effects of Priming and Morph are added (Model 3). With only two factors, the most complex model has two main effects and one interaction, this is Model 1 discussed above. For each subsequent model, the improvement in model fit is evaluated against the cost of introducing extra factors or interactions. In a paper that does not focus on statistical issues not all steps have to be reported in full but a summary of the steps taken to arrive at the final (best fitting)

model should be given.

In a step-wise regression analysis, we look at the R^2 to see whether the inclusion of additional terms improved the model fit of the data. In a mixed model approach, there is no direct equivalent of R^2 and the quantities AIC and -2LL are considered instead. These will be discussed in more detail below, for now, AIC can be viewed as an unstandardized, adjusted R^2 and -2LL will feed into a formal model comparison test, discussed below.

Mixed Model 2 The second model fitted is a so-called null model, which is an intercept-only model without any predictors. In a mixed model analysis, the null model should contain the random factors as described above: Adjustments to the intercept for individual items and individual participants. The model formula is

$$Y_{pi} = \beta_0 + u_{0p} + u_{0i} + \epsilon_{pi}$$

which indicates that the one reaction time obtained for each participant (p) and item (i) combination, is modeled as the intercept β_0 with an adjustment to that intercept for the relative speed of this particular participant u_{0p} , an adjustment for the relative ease of this item u_{0i} , and residual error. Intercept adjustments u_{0s} and u_{0i} both sum to zero, as does ϵ_{ij} , such that the expected reaction time for each observation is the intercept, $E(Y) = \beta_0$.

The results of the null model are summarized in Table 4. The null model had four parameters and resulted in the following fit indices: -2LL=25443, AIC= 25451. There were no fixed effects of interest. Both random effects were significant: for u_{0p} , $Z = 3.90$, $p = .000$; for u_{0i} , $Z = 5.31$, $p = .000$.

This short report on the null model does not include the fixed effect Intercept

and the variance explained by ϵ , as these effects do not aid to our understanding of the model. The reported fit indices will be used as a basis for comparison in the next steps. In an Anova context, the variances of u_{0p} , u_{0i} , and ϵ are summed to compute the contribution of each term to the total variance. In a mixed model context, this is not possible because the u variances are usually correlated.

== TABLE 4 ==

Mixed Model 3 The third model extends the null model with the fixed factors Priming and Morph, but it does not include their interaction. No additional random effects are included in this model, but the existing random effects that adjust the Intercept for participants and items may change due to the inclusion of the new predictors. The model formula is

$$Y_{pi} = \beta_0 + u_{0p} + u_{0i} + \beta_1 \text{Priming}_i + \beta_2 \text{Morph}_i + \epsilon_{pi}$$

which indicates that the one reaction time obtained for each participant (p) and item (i) combination, is modeled as the intercept β_0 , to which there is a participant-specific adjustment u_{0p} and an item-specific adjustment u_{0i} . The predicted RT varies by the factor Priming, with a slope β_1 and by the factor Morph, with a slope β_2 . The word slope is used here for compatibility with the hierarchical linear modeling literature. The expected reaction time for each observation is $E(Y) = \beta_0 + \beta_1 \text{Priming} + \beta_2 \text{Morph}$.

The results of the third model are summarized in Table 5. The model had six parameters and resulted in the following fit indices: -2LL= 25403, AIC= 25415. The fixed effect of Priming was highly significant, $F(1,70)=46.0$, $p=.000$ and so was the effect of Morph, $F(1,1012)=5.39$, $p=.020$. Both random effects

were significant: for u_{0p} , $Z=3.90$, $p=.000$; for u_{0i} , $Z=4.30$, $p=.000$. Compared to the null model (shown in Table 4), the variation related to participants did not change, whereas the variation related to items was reduced substantially. This is to be expected: The fixed factors Morph and Priming should explain some of the variation between items.

== TABLE 5 ==

There are two ways to compare the fit of the null model (Model 2) with the fit of this model (Model 3). First, the values of AIC (Akaike Information Criterion) can be directly compared between the models, with lower values indicating a better fit. The AIC value for Model 3 is 25415, 36 points lower than the value of 25451 obtained for Model 2, indicating an improvement in fit. One cannot determine whether this difference is a significant improvement as AIC values are unscaled (however, as a rule of thumb, a difference of more than 10 points is usually an indicator of a significant improvement.)

A second way of comparing the models is via the likelihood ratio test (LRT). This test uses the raw fit measure *deviance* or *log-likelihood*. Because the likelihood value reported by most programs is log-transformed and multiplied by -2, the abbreviation used in SPSS and SAS is -2LL for *minus two times log-likelihood*. Similar to the F-test, which divides within-variance by between-variance, the likelihood ratio test evaluates the relative fit of Model 3 by dividing it by the fit of Model 2. Division of two likelihoods is mathematically identical to the difference of two log-likelihoods, so we obtain $LRT = 25443 - 25403 = 40$ for the comparison between Model 2 and 3 (the AIC is derived from the -2LL but also takes the number of parameters into account, so the difference in AIC values is similar to the difference in -2LL).

The value of LRT can be statistically evaluated against a chi-squared distribution, using the difference in the number of model parameters as the degrees of freedom. Model 2 has four parameters and Model 3 has six, so a chi-squared with 2 degrees of freedom should be used. The test is $LRT(2) = 40, p < .0001$ which means that Model 3 has a significantly better fit than Model 2. See also Table 6 for an overview of model comparisons.

== TABLE 6 ==

Mixed Model 1, revisited Model 1, presented in Example 1 above, is similar to Model 3, but Model 1 also contains the interaction between Morph and Priming amongst the fixed effects. The random effects are still participant-specific and item-specific adjustments to the intercept.

The results of this model were summarized in Table 3 above. The model had seven parameters and resulted in $-2LL=25399$; $AIC= 25413$. Model 1 has a lower (better) AIC value than the previous models, although the difference with the third model (main effects only) is small and on the edge of significance ($p=.041$) according to the likelihood ratio test.

Using this stepwise procedure, we can conclude that there is some statistical evidence for the presence of an interaction term Morph \times Priming: The interaction term is significant in the F-test presented in Model 1 and Model 1 is a slightly better fit of the data according to AIC values and the LRT test.

Compared to the normal Anova procedure, we have two statistical tests of the interaction at our disposal (F-test and LRT). Because both tests result in p-values that are rather close to our alpha level ($p = .042$ for F-test; $p = .041$ for LRT), it would be wise to investigate this interaction further in a follow-up

experiment or by introducing additional predictors in the design (Keppel & Wickens, 2004), but for now the conclusion of a statistically significant effect of the interaction can be maintained.

Figure 5 shows the syntax for the steps just taken. The null model (Model 2) is specified by removing the PREDICTORS line (or by specifying PREDICTORS=NONE). After constructing Model 3, the models are compared with each other via the COMPAREMODELS command. The output of these commands can be found (slightly reformatted) in Table 6.

In a paper that does not focus on statistical issues, the detailed report of each modeling step given above can be reduced to the findings reported for the final Model 1, including Table 3. The stepwise procedure can be summarized by including Table 6 and a short text such as “A statistical model of the data was built from a null model (Model 2) by stepwise adding all main effects (Model 3), and all interactions (Model 1). In each step, the more complex model showed a significant better fit of the data (see Table 6), leading to the final Model 1.”

== FIGURE 5 ==

Example 3: Contrasts and Post-hoc tests

The examples above have all dealt with one or more factors that each had only two levels (*e.g.*, primed vs. unprimed). If a factor has more than two levels, a test for a significant effect of that factor is usually followed by an examination of which levels differ from each other. Similar to normal Anova procedures, this examination can be done with planned comparisons (also called contrasts)

or omnibus comparisons (also called post-hoc tests).

To illustrate this, the same dataset is used as before, but the two factors Priming and Morph are now combined into one factor Form. Note that the factor Form is constructed for didactic purposes only, this analysis will not clearly distinguish between primed and unprimed words thereby obscuring one of the more important influences on RT.

The new factor Form has three levels: Stem, Inflected and Derived, matching the morphological status of the target word, see also Table 7.

== TABLE 7 ==

Mixed Model 4 This model has one theoretically relevant predictor, the factor Form with three levels. The model contains an intercept and random effects for participant-specific and item-specific adjustments to the intercept.

The analysis yields a model with six parameters and fit indices $-2LL=25395$, $AIC=25407$. The fixed effect of Form was highly significant, $F(2, 1945) = 90.7, p = .000$. The random effects adjusting the intercept were significant: for $u_{0p}, Z = 3.90, p = .000$; for $u_{0i}, Z = 3.11, p = .002$, so inclusion of all random effects was warranted.

Two planned comparisons were run, one comparing the levels Inflection and Derivation, and one comparing the average of these two levels with the Stem form. Both planned comparisons were significant: Derivations versus inflections has a difference estimate of 39.56, $t(1949) = 4.62, p = .000$. Stems versus mean of inflections and derivations has a difference estimate of $-75.43, t(1942) = -12.80, p = .000$.

== FIGURE 6 ==

The DJMIXED syntax for this model is shown in Figure 6. The specification of the fixed and random terms follows the same pattern as before. In Line 7, post-hoc tests are requested, which will be discussed below. Although the post-hoc tests are a theory-free and cautious approach to determining any difference in levels, the application of planned comparisons is more popular. The drawback of using planned comparisons is that any application that is slightly data-driven leads to highly inflated alpha rates. In other words, if the planned comparison was determined after obtaining the means (or preliminary means), the alpha rate is much higher than promised.

The DJMIXED syntax for planned comparison is shown in Figure 6: Line 8 shows how the keyword CONTRAST is followed by the name of the variable, followed by a pipe-symbol (`|`), followed by a specification of the contrast coefficients. Similar to standard Anova contrasts, there should be as many coefficients per contrast as there are levels of the variable. The coefficients in each contrast should sum to zero. The number of contrasts should equal the number of levels minus one, with individual contrasts separated by pipe-symbols. As with standard Anova, using independent or orthogonal contrasts is advisable (Keppel & Wickens, 2004).

A knowledge of the ordering of the levels of the variable is necessary to design and interpret contrasts. SPSS orders levels either numerically or alphabetically, depending on the values of the variable. It is advisable to use a numerical coding with value labels to avoid surprises. Here, numerical values were used and the ordering is stem, derivation and inflection. Note that while the post-hoc output shows the variable labels, the output of planned comparison

does not include this convenience. The two planned comparisons are labeled L1 and L2 in the SPSS output.

Post-hoc tests were requested in line 7, the option is followed by the name of the variable for which post-hoc tests have to be computed. The additional SPSS output caused by this command has two parts: First a table showing mean and standard deviation for each condition, which is useful for creating graphs. Second, six pairwise comparisons are performed that are all highly significant for the current data ($p = .000$ for each, significant after Sidak adjustment for multiple comparison).

Because post-hoc tests involve multiple comparisons, the family-wise alpha has to be controlled. Instead of the familiar Bonferonni *approximation* to the correct alpha for multiple comparisons, the exact formula for alpha correction, as proposed by Šidák, is recommended (see also Abdi, 2007).

Note that all comparisons are based on expected means, not observed means. This implies that a comparison based on a model that does not fit the data well may result in unreliable post-hoc comparisons.

Regression diagnostics and transforms

In both regression and Anova, the distribution of the residuals can inform us about the overall fit of the data and about the specifics of the fit. The DJMIXED package can produce four informative plots: A histogram of residuals, a Q-Q plot of observed versus expected residuals, a detrended Q-Q plot, and a plot of normalized residuals by predicted values. We will only look at the first plot here.

The histogram of residuals for Model 1 is shown in the left panel of Figure 7. This distribution should be close to normal and some appreciable differences exist for this model (the Kolmogorov-Smirnov test is significant, $KS(2004) = 0.106, p = .000$, indicating a significant difference between the observed curve and a normal curve).

On the one hand, the aim of psycholinguistic papers is usually not to provide a perfect fitting model of the data, but to determine whether certain factors make a significant contribution to the prediction of RT or not. Under that view, a moderate to small departure from normality should not overly worry us, although it should be reported. However, ill-fitting residuals can be a sign of a model that does not capture the data very well. The significance of factors and their interactions may be hidden by or caused by the fact that the model does not fit well (see Rouder, Tuerlinckx, Speckman, Lu, & Gomez, 2008, for a promising approach to increasing the model fit of reaction time data).

A commonly suggested transform for reaction time data is the logarithmic transform (Keppel & Wickens, 2004; Van Breukelen, 2005), which will reshape the distribution of RT data so that its heavy right tail is removed and it becomes more akin to a normal distribution. A mixed model analysis of log-transformed reaction times resulted in the same significance levels as Model 1, but improved the distribution of residual (Figure 7, right panel).

Whether the additional fit gained from log-transformation is important should be decided on a case by case basis. Log transforms are not frequently used in the psychology of language literature, but they are common in neighboring fields like cognitive modeling and corpus research. One issue that arises is that a simple log transform ($x_l = \log(x)$) will often turn the fastest RTs into

outliers. The solution is to subtract an estimate of the minimum reaction time, effectively moving the zero point to the right (Rouder et al., 2008): For the log-RT analysis reported in Figure 7, $x_l = \log(x - 100)$ was chosen.

== FIGURE 7 ==

Raw data plots and plots of residuals after fitting intermediary models can also be very instructive as to the structure of the dataset. Textbooks such as Raudenbush and Bryk (2002) and Piñheiro and Bates (2000) show many examples of this. For the current dataset, a plot of the observed data for the primed versus unprimed condition (for derivational pairs only) is shown in Figure 8. In this plot, every item set is represented by a single line. Evidence for word specific adjustments to the intercept can be found at the left edge of the figure, which shows that the item-specific intercepts differ substantially. The statistically significant but not completely compelling interaction effect Morph×Priming may well be due to the heterogeneity of the priming effect on the items: In the figure, three items show negative priming effects (dashed heavy lines) and six items show much stronger effects of priming than the others (solid heavy lines). This could be unsystematic variation of the efficacy of Priming, but facing such data, it is wise to investigate whether there are any factors which may help explain this.

== FIGURE 8 ==

More complex models

In almost every case, the stepwise analysis presented above should be sufficient to draw psycholinguistically valid conclusions from the data. The mixed model

can be further extended in two ways, which will be briefly outlined here.

The first extension concerns covariates: On some occasions, there are known covariates that may help explain the differences between items (or more rarely, participants). If there are theoretical reasons to expect that, for example, log frequency will co-determine reaction times, this predictor should be included in the model. The mixed models described here are very similar to normal regression models and an effect of frequency can be added as a predictor in a straightforward way. The model formula and the DJMIXED syntax are included in the online Appendix.

The second possible extension concerns the way differences between items and participants influence the expected outcomes. In the models so far, the random effect of items (and participants) has been added to the intercept to indicate whether an item is *generally* easy or hard. It is possible that the effect of a predictor (say, Priming) also differs between items (or between participants). One way to account for that is to include a second random effect for items, which modifies the strength of the effect of Priming. Mathematically, a new random effect u_{1i} is added to the β for Priming, as shown in this partial formula: $Y_{ij} = \dots + (\beta_1 + u_{1i}) \cdot PrimeType_i + \dots$. In mixed model parlance, the random effect u_{1i} modifies the slope (β) of Priming.

A number of statistical complications arise with this type of analysis and the online Appendix goes into some detail on how to work around these. However, there are further reasons why this type of analysis may not be applicable to most psycholinguistic experiments.

First of all, the exact structure of the random effects is rarely a psycholinguistic goal in itself. A model with a random effect on the slope of Priming does not

give a better theoretical explanation, it merely adds a device for capturing unexplained variance. A well-chosen covariate is often a better option, as it does add theoretical strength to the model.

Second, extracting three or more random effects from the data is demanding. Psycholinguistic data tend to have one observation per participant-item combination and the numbers of items and participants tested are sizable, but not in the 100s. Both of these factors limit the ‘carrying capacity’ (Nezlek, 2008) of the data. The two random effects related to items (u_{0i} and u_{1i}) are most often correlated, which makes it harder to arrive at estimates for each of them, necessitating large number of items and participants.

Third, the extended analysis assumes that the item-related random effects modifying the intercept and the slope of Priming are independent influences, which may be correlated. As has been argued by Rouder et al. (2008), faster items often show less variability and are therefore inherently less sensitive to priming. In a sophisticated model that creates a connection between an item’s mean and its standard deviation, these authors were able to show that there was no need for item or participant-specific adjustment to slopes once the correlation between mean and standard deviation was taken into account.

In sum, there seem to be few compelling reasons to add random effects that modify slopes. For psycholinguistic data, models like those presented above already provide a better description of the data than classical Anova models and it may well turn out that the extra complications caused by adding random effects modifying slope are rarely necessary in practice. Authors should run the usual regression diagnostics to determine whether the data was fitted reasonably well or whether further statistical explorations are necessary.

Discussion

This paper has presented a simple framework to address the issue of random participants and random items in language experiments. The DJMIXED extension to SPSS should put this mixed models analysis within the reach of every psycholinguist. It was argued that the results of mixed models are easy to interpret while staying much closer to the data than the other approaches that are currently in common use (min F' , F_1/F_2 , treating items as fixed).

Mixed models should only be used when the dataset is large enough and after outliers and wrongly coded observations are removed. Conceptually, a separate regression line is estimated for each level of Participants and also for each level of Item, so an extreme outlier can have a large influence if the number of observations per participant or the number of observations per item is low. Compared to an Anova, the restrictions on the data imposed by mixed modeling are very relaxed, as missing data and unequal cell sizes are not a problem and homoscedascity is not an a priori requirement either. Mixed models require equality of residual variance, that is, the predictors should not only capture the difference in average RTs but also any difference in variability of RTs. For most datasets, this seems a tenable assumption (but see Rouder et al., 2008) and there are currently few alternatives for those cases in which this assumption is mildly violated.

Mixed models are a relatively recent extension to the statistical canon and although the pace of development has slowed down, further improvements to these models and their evaluation will most certainly be found. However, the methods of model evaluation that are suggested here (F-tests and LRT) have

shown their merits outside of mixed-modeling, and they are implemented in major statistical packages like SPSS and SAS, and are generally recommended in various fields of science.

Like most statistical tests, these tests are not perfect under all circumstances: Using the LRT to test for the inclusion of random effects is slightly conservative when the difference between parameters in the two models is used as the degrees of freedom (Stram & Lee, 1994; Kreft & De Leeuw, 1998). The alternative of using a 50/50 chi-squared mixture was suggested by Stram and Lee (1994) and is adopted in the appendix to this paper and elsewhere (Kreft & De Leeuw, 1998; Verbeke & Molenberghs, 2001). But criticisms against this procedure have been leveled (Piñheiro & Bates, 2000, p. 70; Baayen et al., 2008), suggesting it may still be slightly conservative (not rendering enough significant results). Faraway (2006) suggests to use a parametric bootstrap (cf. Janssen, Bickel, & Zúñiga, 2006) to correct the p -values of the LRT when testing for the inclusion of random effects and this procedure has the advantage over the solution suggested by Baayen et al. of not depending on a Bayesian framework.

LRT tests for the inclusion of fixed effects are also widely used (Kreft & De Leeuw, 1998; Verbeke & Molenberghs, 2001; Raudenbush & Bryk, 2002), but some have argued that this test may be too liberal (allowing too many significant results) for tests on certain datasets (Piñheiro & Bates, 2000, p. 88; Baayen et al., 2008). The example given by Piñheiro and Bates cautions the reader not to test for the inclusion of fixed effects with a very large number of levels compared to the total number of observations. It is shown that the LRT can be too liberal when testing for the inclusion of a factor with 15 levels in a dataset with only 60 observations. In practical psycholinguistic

applications, the number of levels of a fixed factor hardly ever exceeds five so if the general recommendations for sizable numbers of participants and items, and therefore observations, are followed, this criticism should not overly concern us. Raudenbush and Bryk evaluate the merits of the LRT compared to a multiple comparison procedure similar to the contrasts discussed above and conclude the LRT is a valid procedure that will produce results nearly identical to multiple comparison (Raudenbush & Bryk, 2002, p. 61), while the LRT is much easier to implement. When comparing models for the inclusion of a fixed effect, the multiple comparison tests are similar to the F-test that was used here in conjunction with the LRT.

SPSS and SAS report z -tests on individual model β s, which should not be used for drawing conclusion about the importance of predictors. The z tests can be very conservative and Raudenbush and Bryk (2002) suggest to use a t -distribution instead. The issue was side-stepped here by using F-tests (technically, Type-3 F-tests) of fixed effects instead. In this paper, values of β were reported in tables to offer the reader an insight in the direction and magnitude of the effect, but the p -values listed are derived from the F-tests on the fixed effects in the analysis. The advantage of the omnibus F-tests is that they are available in SPSS and in SAS and that they are similar in interpretation to the normal Anova tests. The F-tests also produce one significance value for factors with more than 2 levels, whereas multiple significances result if the z -tests are followed (one z -test is presented for each β). This technique is followed by almost every text on hierarchical linear modeling and mixed modeling (Faraway, 2006; Hox, 1995; Kreft & De Leeuw, 1998; Raudenbush & Bryk, 2002; Singer, 1998; Snijders & Bosker, 1999; Verbeke & Molenberghs, 2001).

F-tests (including Type-3 F-tests) for normal Anova and mixed-models alike have been criticized by Venables (1998). To evaluate the significance of fixed effects, it has been suggested to askew F-tests (Baayen, 2008; Bates, 2006, 2008) and use MCMC (Monte Carlo Markov Chains), a simulation technique based on Bayesian principles to approximate the significance of each fixed effect on an analysis-by-analysis basis. This technique has certain theoretical advantages for data with smaller numbers of cases, but it is not implemented in SPSS or SAS. It also requires one to work within a Bayesian inference framework, which has various advantages and disadvantages that fall outside of the scope of this paper. In a discussion of which test to use, Faraway (2006, 2009) recommends the combined use of the F-test and the LRT.

Of course, statistics is a scientific discipline just like psycholinguistics, and dissenting opinions, alternative approaches and progressing insights are par for the course. Mixed models, and hierarchical linear models as their special case, are a mature technique and they have been implemented in the major statistical packages since 1996 (SAS), 2000 (R), and 2002 (SPSS). Straightforward and relatively uncomplicated applications of mixed models, such as advocated in this paper, are used in biology (O'Connor et al., 2007), educational research (Raudenbush & Bryk, 2002), social psychology and personality research (Nezlek, 2008), signal detection theory (Rouder & Lu, 2005), and many other fields. Mixed models are easy to construct in SPSS and SAS and the mixed model results are straightforward to understand when the focus remains on the fixed effects. It is time for psycholinguistics to leave the realm of F_1/F_2 testing and move to mixed modeling as a standard means of assessing significance.

Acknowledgements

I would like to thank Nicolas Dumay and Lea Hald for constructive comments on an earlier draft. Thanks to my former colleagues at the University of Kent (and especially Joachim Stoeber) for their continuing support. This paper has been shaped by a number of workshops on mixed modelling that I have given, thanks to all participants for clarifying to me what makes mixed modelling so hard to understand.

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Table 1

Dummy coding for a predictor with four levels and corresponding expected reaction times, $E(Y)$.

	Dummy			
PrimeType	PTB	PTC	PTD	$E(Y)$
A	0	0	0	β_0
B	1	0	0	$\beta_0 + \beta_1$
C	0	1	0	$\beta_0 + \beta_2$
D	0	0	1	$\beta_0 + \beta_3$

Table 2

Data for Example 1, average reaction times (ms) and standard deviations.

		Morph	
		derivation	inflection
Priming	unprimed	683 (172)	646 (162)
	primed	594 (141)	588 (140)
Difference		89	58

Table 3

Fixed and Random effects for Model 1, Example 1

Fixed Effects				
Model term	Category	β	F	p
Priming	Unprimed	59.55	47.9	.000
Morph	Inflected	8.29	9.2	.003
Priming×Morph	Unpr+Infl	32.44	4.2	.042
Random Effects				
Model Term	Adjustment for	Variance	Z	p
u_{0p} Intercept	Participants	5144	3.90	.000
u_{0i} Intercept	Words	1774	4.27	.000
ϵ Error	–	16880	30.55	.000

Table 4

Random effects for Model 2, the null model

Random Effects					
Model Term	Adjustment for	Variance	Z	p	
u_{0p}	Intercept	Participants	5129	3.90	.000
u_{0i}	Intercept	Words	3507	5.31	.000
ϵ	Error	–	16861	30.61	.000

Table 5

Fixed and Random effects in the third model, main effects only

Fixed Effects					
Model term	Category	β	F	p	
Priming	Unprimed	75.5	46.0	.000	
Morph	Inflection	16.7	5.4	.020	
Random Effects					
Model Term	Adjustment for	Variance	Z	p	
u_{0p}	Intercept	Participants	5142	3.90	.000
u_{0i}	Intercept	Words	1858	4.30	.000
ϵ	Error	–	16891	30.6	.000

Table 6

Overview of Models 1 to 3, with the degrees of freedom (df), deviance (-2LL), and the Akaike fit index (AIC). Models are compared with the likelihood ratio test (LRT), a significant results indicates that the more complex model is preferable. For each comparison, base for comparison, the comparison *df* and LRT value are shown. All models contain random adjustments to the intercept for participants and items (u_{0p} and u_{0i}).

		Model details			Likelihood Ratio		
	Fixed effects	df	-2LL	AIC	Comparison	df	LRT
2	None	4	25443	25403	–	–	–
3	Priming, Morph	6	25403	25415	2	2	39.92***
1	Priming * Morph	7	25399	25413	3	1	4.16*

Note: * = $p < .05$; ** = $p < .01$; *** = $p < .001$

Table 7

Relationship between the factors Priming, Morph and Form

Factor Prime	Description	Factor Morph	
		Inflection	Derivation
Unprimed	Presented First	inflectional form	derivational form
Primed	Presented following related inflection	stem form	—
	Presented following related derivation	—	stem form

Figure Captions

Figure 1: Left panel: Reaction times for 60 hypothetical items in unprimed and primed conditions, each item has an individual adjustment (u_i) to reflect its inherent speed. Right panel, distribution of adjustment values u_i is close to normal.

Figure 2: DJMIXED point-and-click interface

Figure 3: DJMIXED syntax for Example 1

Figure 4: SPSS syntax for Example 1

Figure 5: DJMIXED syntax for remaining models and model comparison (line numbers added)

Figure 6: Syntax for mixed model 4 with specification of post-hoc tests and contrasts, line numbers added

Figure 7: Histogram of residuals (with normal curve superimposed) for models with the dependent variable RT (left panel) and log-transformed RT ($\log(RT - 100)$), right panel).

Figure 8: Plot of observed primed and unprimed reaction times for each derivational pair, with negative priming (dashed lines) and strong positive priming (solid lines) highlighted.

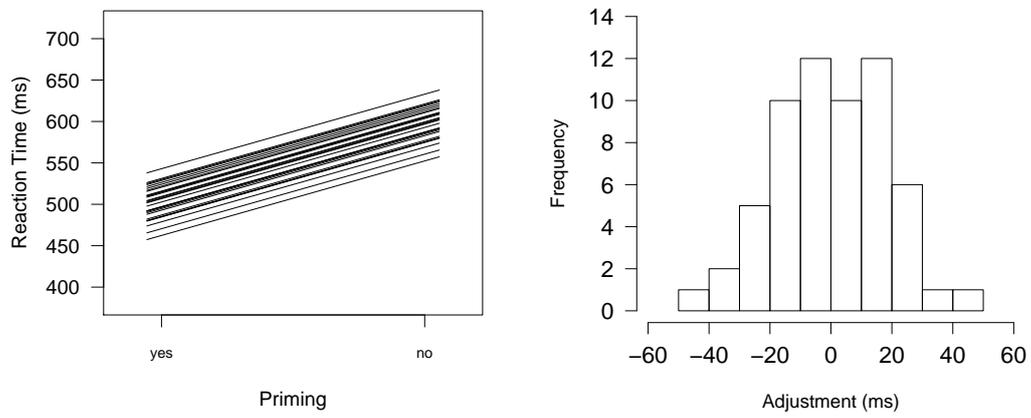


Figure 1.

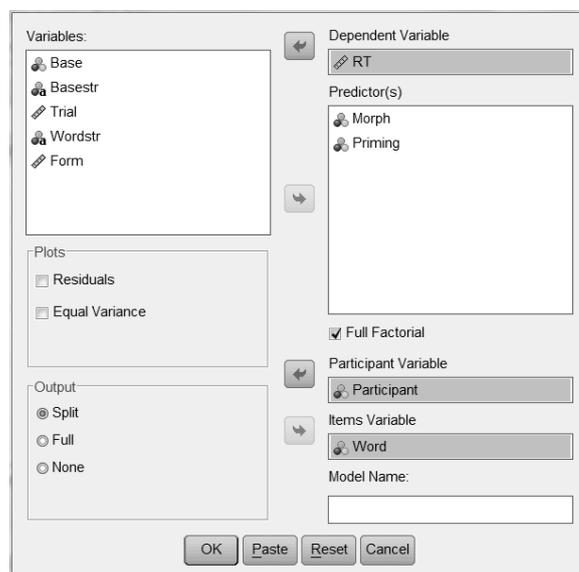


Figure 2.

```
1  * SPSS syntax when using the extension package .
2  DJMIXED /MIXEDMODEL
3      DV = rt
4      PREDICTORS = priming morph priming*morph
5      PPS = Participant
6      ITEMS = Word
7      NAME = 'interaction' .
8  * Produces a short summary .
9  DJMIXED /MODELSUMMARY
10     NAME = 'interaction' .
```

Figure 3.

```
1  * SPSS syntax for Model 1: fixed effects and interaction .
2  MIXED rt BY priming morph
3  /FIXED= priming morph priming*morph
4  /RANDOM=intercept | SUB(Participant) COVTYPE(VC)
5  /RANDOM=intercept | SUB(Word) COVTYPE(VC)
6  /METHOD=ML
7  /PRINT=COVB SOLUTION TESTCOV
8  /CRITERIA=CIN(95) MXITER(5000) MXSTEP(50) SCORING(1)
9  SINGULAR(0.000000000001) HCONVERGE(0,
10  ABSOLUTE) LCONVERGE(0, ABSOLUTE)
11  PCONVERGE(0.000001, ABSOLUTE)
```

Figure 4.

```
1  * syntax for the null model (Model 2).
2  DJMIXED /MIXEDMODEL
3      DV=rt
4      PPS=Participant
5      ITEMS=Word
6      NAME='null' .
7
8  DJMIXED /MIXEDMODEL
9      DV=rt
10     PREDICTORS = morph priming
11     PPS=Participant
12     ITEMS=Word
13     NAME='main effects' .
14
15  DJMIXED /COMPAREMODELS
16     NAME1='null'
17     NAME2='main effects' .
18
19  DJMIXED /COMPAREMODELS
20     NAME1='main effects'
21     NAME2='interaction' .
```

Figure 5.

```
1  DJMIXED /MIXEDMODEL
2      DV = rt
3      PREDICTORS = form
4      PPS = Participant
5      ITEMS = Base
6      MODEL = 'form as pred'
7      POSTHOC = form
8      CONTRAST = form | 0 1 -1 | 1 -0.5 -0.5 .
```

Figure 6.

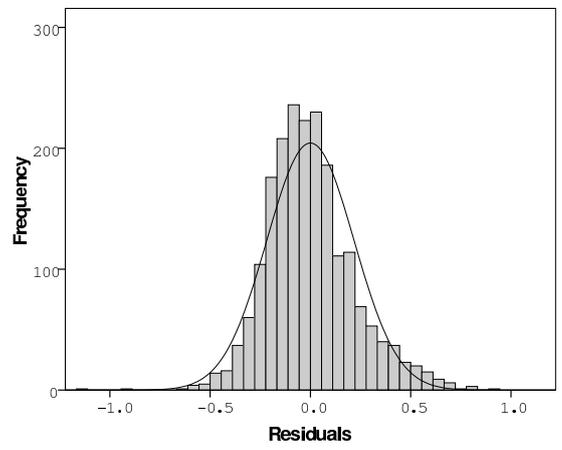
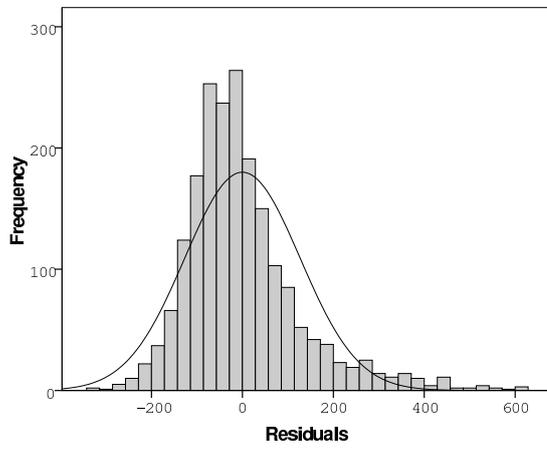


Figure 7.

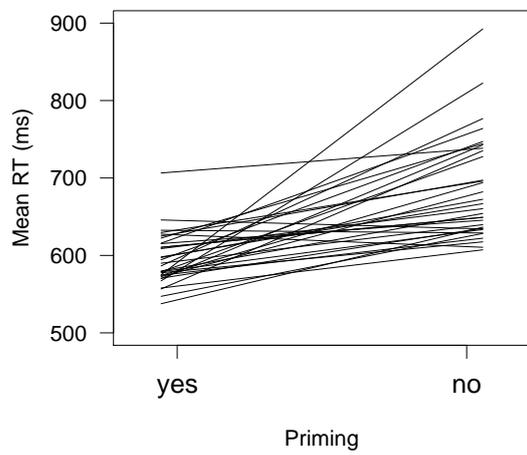


Figure 8.